RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2012

SECOND YEAR

Date : 22/12/2012 **MATHEMATICS (General)**

Time : 10.30 am – 1.30 pm Paper : III Full Marks : 75

[Use separate Answer book for each group]

Group – A

*Answer **any four** questions* : [4x5=20]

1. a) Find the point where the line joining (2,0,4) and (–4,–2,–6) is divided by the xy-plane.

   b) Prove that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. [2+3]

2. a) Obtain the equation of the plane containing the line $\frac{x}{a}+\frac{z}{c}=1$, y=0; and parallel to the line $\frac{y}{b}-\frac{z}{c}=1$, x = 0.

   b) Find the direction cosines of the normal to the plane 2x + 3y + 4z – 4 = 0. [4+1]

3. Prove that the straight lines $\frac{x-1}{3}=\frac{y+2}{1}=\frac{z+3}{2}$ and $\frac{x-5}{7}=\frac{y+8}{-5}=\frac{z-6}{11}$ are coplanar and find the equation of the plane on which they lie. [2+3]

4. Find the shortest distance between the straight lines $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ and $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$. Find also the equation of the line of shortest distance. [3+2]

5. A sphere of constant radius r passes through the origin and cuts the axes in A, B, C respectively. Prove that the locus of the perpendicular drawn from O to the plane ABC is $(x^2+y^2+z^2)^2(x^{-2}+y^{-2}+z^{-2})=4r^2$. [5]

6. Find the equation of the right circular cone whose vertex is (-1, 2, 3), the semi-vertical angle is 60° and the axis is parallel to $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$. [5]

Group – B

*Answer Question No. 7 and **any two** from the rest* : [8+32=40]

7. Answer **any four** questions : [4×2]

   a) Draw graphically the feasible space given by the L.P.P.

   Maximize Z = $2x_1 + x_2$

   subject to $x_1 \le 2$

   $x_2 \le 3$

   $x_1 + x_2 \ge 1;\ x_1, x_2 \ge 0$

   b) What is the difference between a basic solution and a degenerate basic solution?

   c) Define a convex set with an example.

   d) Express the given L.P.P in standard form :

   Min Z = $2x_1 - x_2 + x_3$

   Subject to $2x_1 - x_2 + x_3 \le 1$

   $x_1 - 2x_2 + x_3 \ge 2$

   $x_2 + x_3 \le 3$

   $x_1, x_2, x_3 \ge 0$

e) Find the initial basic feasible solution by North-West Corner Method for the following transportation problem :

	D_1	D_2	D_3	a_i
O_1	5	1	8	12
O_2	2	4	0	14
O_3	3	6	7	4
b_j	9	10	11	

f) State the Fundamental theorem of L.P.P.

g) State why an assignment problem is not an L.P.P.

8. a) Solve graphically the following linear programming problem :

Minimize $Z = 2x - y$

Subject to $x + y \le 5$,

$x + 2y \le 8$

$4x + 3y \ge 12$

$x, y \ge 0$

b) Three different types of lorries A, B and C have been used to transport 60 tons solid and 35 tons liquid substance. A type lorry can carry 7 tons solid and 3 tons liquid, B type lorry can carry 6 tons solid and 2 tons liquid, C type lorry can carry 3 tons solid and 4 tons liquid. The costs of transport are Rs. 500·00, Rs.400.00 and Rs. 450·00 per lorry of A, B and C respectively. To find the minimum cost, formulate the problem mathematically.

c) Find the basic feasible solutions of the system of equations

$x_1 + x_2 + x_3 = 8,$

$3x_1 + 2x_2 = 18,$

$x_1, x_2, x_3 \ge 0.$ [6+6+4]

9. a) Show that a hyperplane in E^n is a convex set.

b) Find the extreme points, if any, of the following set after proving it to be a convex set :

$S = \{(x, y) \in E^2; |x| \le 1, |y| \le 1\}$

c) (2,1,3) is a feasible solution of the set of equations

$4x_1 + 2x_2 - 3x_3 = 1,$

$6x_1 + 4x_2 - 5x_3 = 1.$

Reduce it to a basic feasible solution of the set. [4+6+6]

10. a) Solve Graphically the following L.P.P.

Minimize $Z = 2x_1 + 3x_2$

Subject to $2x_1 + 7x_2 \ge 12$

$x_1 + x_2 \ge 6$

$5x_1 + x_2 \ge 10$

$x_1, x_2 \ge 0$

b) Use Vogel's Approximation method to obtain an initial basic feasible solution of the transportation problem : [8+8]

2	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	

11. a) Find out the optimal assignment cost from the following cost matrix

18	26	17	11
13	28	14	26
38	19	18	15
19	26	24	10

b) Solve the L.P.P by simplex method

Maximum $Z = x_1 + x_2 + 3x_3$

Subject to $x_1 + 2x_2 - x_3 \le 10$

$4x_1 + 3x_2 + 2x_3 \le 8$

$x_2 + 3x_3 \le 15$

$x_1, x_2, x_3 \ge 0$

c) Show that a transportation problem can be written as an L.P.P. [6+6+4]

Group – C

*Answer **any three** questions* : [3x5=15]

12. a) What is meant by absolute and relative errors?
If $u = 3v^7 - 6v$, find the relative error in u at v = 1 if the error in v is 0·05.

b) Round-off the following number to four significant figures as well as four decimal places : 38·46235. [3+2]

13. From the following table, find f(1·02) using Newton's Forward interpolation formula : [5]

x	1·00	1·10	1·20	1·30
f(x)	0·8415	0·8912	0·9320	0·9626

14. Use the method of bisection to find the root of the equation $x^4 + 2x^3 - x - 1 = 0$ lying in the interval (0,1) correct to two significant figures. [5]

15. Evaluate $\int_1^3 \frac{1}{x} dx$ by Simpson's $\frac{1}{3}$ rule using eight equal subintervals. Determine the error by direct integration. [4+1]

16. Using appropriate interpolation formula find the value of f(5) from the following data : [5]

x	3	4	6	8
f(x)	4·5	13·2	43·7	56·4